unrelated numbers may be plausibly regarded as merely the probable deviations of random sampling.

Another line of evidence tending to show the fortuitous character of the occurrences relates to the probability of the frequency of the intervals from crest to crest or hollow to hollow in a series of unrelated numbers. This serves as a criterion for testing the conformity of meteorological data to the requirements of fortuitous occurrence. According to Besson, in a series of unrelated numbers the number of two-intervals is greater than the number of three-intervals in the ratio of 40 to 33. A large number of series of annual means of temperature and pressure at stations in Europe and the United States were examined and practically all show a decided preponderance of the three-year interval.

Dr. Brooks's later conclusions, therefore, that the sequences of winter temperatures, particularly as regards alternations of warm and cold, are mainly what would be expected from chance occurrence seems to be borne out by the evidence presented above.

A POSSIBLE RAINFALL PERIOD EQUAL TO ONE-NINTH THE SUN-SPOT PERIOD.

551.578.1 : 551.590.2

By DINSMORE ALTER, Ph. D.

[University of Kansas, Lawrence, Kans., Oct. 1, 1920.]

SYNOPSIS.

The first search for a rainfall periodicity was based on a hypothetical constant periodicity equal to Turner's earthquake period which is exactly one-ninth of the mean sun-spot period. The results were inconclusive, but indicated that a further search might be worth while.

Next a graphical method was devised by which it was possible to vary the length of this periodicity, keeping it always in step as the ninth harmonic of the varying sun-spot period. The results obtained were much more conclusive.

All the sectional rainfall averages for each of the 42 sections of the United States were examined and tables and curves showing the results for each of three parts of the United States (Eastern, Central, and Pacific) are appended to the paper. The first half and the last half of the data were for the most part considered separately and a fair similarity obtained from these stretches of independent data. Curves from these separate data are plotted together in order to show plainly points of resemblance and of dissimilarity. The reader must judge for himself whether these resemblances can be accidental or must be due to a real periodicity.

The work is being continued and it is proposed to examine the data of other countries and to search for other possible harmonics of the sun-spot period. An examination of temperature data is also planned.

INTRODUCTORY.

In August, 1915, Dr. A. E. Douglass read a very interesting paper before the Berkeley meeting of the American Astronomical Society regarding an investigation of the growth of trees in many parts of the world, indicating an 11-year period in rainfall (1).

It seemed to me that the data collected by the Weather Bureau should definitely settle such a question of periods. Some preliminary reading showed, however, that a tremendous amount of time had been spent on the problem (2) and that if solvable it must be very complicated. Other work prevented starting any actual investigation; then the war intervened and the problem was untouched till the spring of 1919. The first data examined were those from Lawrence, Kans., where records since 1868 are available. Several hundred hours of work showed nothing. Once a stretch of five years was found which resembled another five quite closely after eliminating the seasonal curve. Another time resemblances were found after about 22 years. All such were easily explainable as accidental. It seemed useless to carry the work further with the data at hand.

A paper by Prof. Turner (3), however, gave me a new suggestion, although there was little if any logical reason for any connection. In this paper, Prof. Turner shows plainly the existence of a period in earthquakes with a length between 14.8421 and 14.8448 months. It occurred to me that this period might be commensurable with the sun-spot period. Upon multiplying it by 9 I obtained 11.13 years, which is the mean sun-spot period to the exact hundredth of a year. Such an exact coincidence is very probably not accidental (4).

My next move was to examine all sun-spot data in order to find whether such a period also exists in sunspots as the ninth harmonic of the primary period. The preliminary results based on a constant period equalling one-ninth the mean sun-spot period were inconclusive though possibly favoring the existence of the period. After the investigation of the rainfall data the problem was attacked again in a different manner and led to a much more positive result. This will be discussed more fully in a later paper. The idea now came to me to investigate the rainfall data by the same method that Prof. Turner has used on the earthquake data. This method is available only after some extraneous idea has indicated at least an approximate period to be investigated.

MATERIAL SUITABLE FOR HARMONIC ANALYSIS.

A mass of observational material when plotted with time as abscissae and observed values as ordinates may show no repetition of the same curve, even though such a curve might exist. There may be nothing definite about it to indicate a period. In such cases ordinary methods of harmonic analysis become useless. This failure to repeat values, when a period exists, may be due to any one or more of the four following causes:

(*a*) Incommensurable periods may coexist. In this case the curve will never repeat itself, although for short periods of time there may be a fairly close approximation to such repetition. If there are three or more incommensurable periods the curve obtained for the data is very complex. For example, the seasonal variation of the rainfall would be incommensurable with a possible one equalling the sun-spot period. Of course, if one of such periods is known, as in the case of the seasonal variation in the example above, it may be eliminated.

(*b*) There may be large accidental errors. Such errors mask a periodicity almost completely in any one cycle and disappear only when the data values in each of a number of well-distributed phases are added through many cycles. From the theory of errors their influence will be inversely proportional to the square root of the number of cycles added.

(*c*) Long-period variations may exist. If there are periods longer than the interval of the data they will produce much the same effect as accidental errors or incommensurable periods.

(*d*) There may be periods which vary in length. An example of such a period is the sun-spot period, which, although averaging 11.13 years, has varied from 7.3 to 17.1 years during the last 115 years.

When any one of these four difficulties exists it is almost (**10**) impossible successfully to treat the problem unless the investigator stumbles upon the true period either by a fortunate suggestion, by some reason extraneous to the problem, or by the patient trial and error method by which Kepler found his three laws of planetary motion.

METHOD USED BY TURNER IN EXAMINING THE EARTHQUAKE DATA.

The exact form of this method is due to Schuster (**7**) and is a slight modification of the one astronomers use in examining the radial velocities of spectroscopic binaries and other data. Suppose that we have a mass of material—for example, the number of earthquakes recorded per month, or the rainfall per month—through many years. Plotting shows no periodicity, or at the most only a faint hint of such. Some chance leads us to suspect a period of, for example, 15 months. We can write the first 15 months' data in a row as the heads of as many columns. The sixteenth month, the thirty-first, etc., will follow successively in the first column, the seventeenth, thirty-second, etc., in the second column, and so on to the thirtieth, forty-fifth, etc., in the fifteenth column. Each column will then contain only months which are in the same phase of the suspected period if it actually exists.

We will refer to one such row as a cycle, and to the columns as phases. Suppose the period to exist. It may not show in a single cycle, probably will not, because of large accidental errors or incommensurable periods, either or both of which may be present. But the months of any phase of an incommensurable period will, in the long run, be almost evenly distributed through all the phases of our assumed period, and will, therefore, be subject to the same laws as accidental errors; namely, their influence will be inversely proportional to the square root of the number of cycles. In the course of four cycles (five years in our present example), their importance will be only half as great as for any one cycle; after 16 cycles one-quarter as great, etc. However, the effect of our assumed 15-month period will be equal in each period and, therefore, as prominent in the average as in any one cycle. Thus, no matter how large the accidental errors, or the variation due to incommensurable periods, the true variation from phase to phase will begin to appear. If the assumed period does not exist the mean values of the phases will approach each other as we increase the number of cycles.

This last point gives us two very powerful criteria for the verity of our assumed period:

(*a*) Having given a large number of cycles, we may compare the phase values of the first half of the cycles with those of the latter half. If the variation be real the curves from the two halves of the data should agree fairly well. If the variation be accidental there can be only chance resemblance. Unless the assumed period exists the two halves of the data are entirely independent if there are enough cycles to eliminate residuals of other periods that might exist.

(*b*) Having obtained the phase values as above for each half of the data, we may consider half the difference of identical phases in the first and last halves of our data as a measure of the deviation of the two curves from each other and of the amount of chance error left in each phase. Call this half difference d. We will have in this example $d_1, d_2 \ldots\ldots d_{15}$. The probable error of any point on the curve which is formed from the whole of the data will be given by the formula.

$$\epsilon = 0.6745\sqrt{\frac{\Sigma(d^2)}{n-1}}$$

If this probable error is as large as half the variation from maximum to minimum phase there is an appreciable chance that the variation is accidental. If it is smaller than about *one-fourth* the variation, the chances are less than one in a thousand that it is accidental. Both these criteria must be applied in any case under discussion.

Let us suppose that the assumed period is not an exact number of months; for example, 14$\frac{6}{7}$ months. In this case seven cycles will equal 104 *instead of 105 months*. We must spread our 104 months over seven cycles of 15 phases each; that is, over 105 phases. To do this we will fill each of the first six cycles and the first 14 phases of the seventh cycle just as formerly, using all the data that we have for seven cycles. We will then use the month's data which we used for the 14th phase of the 7th cycle again in the 15th phase. Doing this, no month will fall more than a half phase from the proper one as determined by the mean of all positions. If we assume a period of 15$\frac{1}{7}$ months we will merely skip one of the month's data, or, better still, average it with the two immediately adjacent. In this manner any period may be plotted with any number of phases desired and no month's data more than a half phase from its proper place.

FIRST APPLICATION OF THIS METHOD TO RAINFALL.

One-ninth of the mean sun-spot period is very nearly 14$\frac{4}{7}$ months. I tabulated all the rainfall data from Lawrence, Kans., beginning 1868, *according to the* method outlined above. The result showed a variation of about 12 per cent each side of the normal. Next I divided the data into halves and found the two to agree fairly well. Following this I examined data from all of Kansas, from Nebraska, New England, and Ohio. The data from Ohio checked fairly well; those from New England and Nebraska gave results which were discordant with themselves. The variation of the sun-spot period now came to mind. If there were any real variations due to sun-spots or to a common cause it would certainly have to keep a constant relationship with the phases of the sun-spot period.

Table I shows the dates of maxima and minima of sun-spots as determined by Wolf and Wolfer (**5**). It also shows the number of years intervening between successive maxima or minima; in other words, the actual sun-spot periods during those years. As a first approximation to keeping the phases in step with the sun-spots I plotted the rainfall between the dates of each pair of consecutive minima on a period of one-ninth that interval. For example, minima occurred 1889, August and 1901, September. The interval is 145 months. I therefore used a period of 16$\frac{1}{9}$ months between these dates. The next minimum occurred 1913, May. This interval is 141 months, and I used a period of 15$\frac{2}{3}$ months between these dates. When this was done I secured very much better results than before, so much better that I could not believe them due to accident. I obtained similar curves for each State the whole length of the Atlantic and Gulf coasts as far as Texas. When the data of New England and Pennsylvania were divided in halves, curves similar in shape were obtained for each, differing only in phase. This improvement over the results from a constant period indicated that a more rigid method of keeping constant relationship with the sun-spot phases should be devised before definite conclusions were drawn.

RIGID FOLLOWING OF THE SUN-SPOT PHASES.

It is evident that the sun-spot period between the minima named above had values of 145 and 141 months, respectively. Let us examine the two maxima occurring within these dates. One occurred 1894 February and the other 1906 May with an interval of 147 months. This must have been the value of the sun-spot period between these dates. It is longer than the period obtained from either pair of minima named above, yet it occurs as part of each of them and contains no part that is not in one or the other of them. We are forced, therefore, to the conclusion that if continuous (**6**)—

The length of the sun-spot period is continuously varying (**2**) *and a value of the period obtained between successive maxima or successive minima is merely an average of all values passed through in this interval.*

If we had a curve with time plotted along the axis of abscissae and the corresponding values of the sun-spot period as ordinates, the average value of the sun-spot period between the maxima or two minima occurring at t_1 and t_2 would be given by:

$$t_1 - t_2 = \text{Average value} = \int_{t_1}^{t_2} \frac{\text{curve}}{t_1 - t_2}$$

If we plotted abscissæ and ordinates on the same scale, these average values would form squares bounded by ordinates through the dates which limited them. The area between the axis of abscissæ and the unknown curve described above, representing the actual value of the period at any date, in the interval between two maxima or two minima would have to equal the corresponding known square. Since these squares overlap we know the values of a series of overlapping definite integrals of the unknown curve. From these data it is possible, assuming the simplest curve to be the true one, by the aid of a planimeter, to construct the curve without knowledge of its mathematical form. In doing this it is easier to choose some convenient period as the axis of abscissæ and to measure departures from this period. Changing the axis in this way merely changes all the integrals by a known constant amount and changes the known squares into known rectangles. It is also practical to magnify the scale of ordinates very much over the scale of abscissæ. Locating the curve consists first in measuring the area of each of the rectangles; then penciling in what appears to be the curve, measuring the definite integrals of the approximate curve with the planimeter; erasing for a new approximation and repeating many times. In the curve of sun-spot values reproduced as figure 8 I have erased each part of the curve probably a hundred times. Although very laborious, the process with enough patience yields very good results. The accuracy of the period curve depends upon the accuracy with which the epochs of maxima and minima are obtained. A steep, but narrow, peak, such as that of 1861, may be unreal for this reason. However, due to the short duration of such a peak and the fact that it must almost immediately be counterbalanced, there will be little effect in data extending over a long range.

In the preceding paragraph I have spoken of the sun-spot period at any date as a varying quantity not even approximately constant through a single cycle. This may necessitate a definition of "period" somewhat different from what is ordinarily understood. I therefore give the following definition which will be adhered to whether referring to sun-spots or rainfall.

The period at any date is the rate of change of phase at that date and need not continue even approximately through a complete cycle.

From this curve I have taken the mean value of the sun-spot period for each year. These values are given as column 2 of Table 2. Column 3 gives the departures from 15 months of one-ninth these values. Obviously, 15 months was chosen because it is the nearest integral number of months to one-ninth the mean sun-spot period. The unit for column 3 is one-ninth of a month. If, for example, the number given for any year in column 3 were +9, it would mean that during that year one-ninth of the sun-spot period was 16 months. If it were −9 it would mean that this period was 14 months. In the first case it would be necessary, working on a 15-phase basis, to skip a month every 16 months as long as the period persisted; in the second case to repeat one every 14 months. We can thus construct a table of months to be repeated in the analysis of our rainfall data when the ninth of the sun-spot period is less than 15 months, or to be skipped (or better still, averaged with the two adjacent ones) when the ninth is more than 15, in order that Wolfer's sun-spot maxima may all fall in one phase and his sun-spot minima is one.

In this work I have in each case averaged the month to be skipped with the months immediately adjacent, instead of actually skipping. Thus three months' data give two phases, the result desired through skipping, and all data is used. There is, however, such a slight gain in accuracy that I do not believe it worth the slight extra work involved. If this is done the maxima of all the cycles of the sun spots will always fall in one phase of the suspected rainfall variation and also the minima in one. Wolfer's values of maxima and minima are uncertain by a month or so, and I have therefore considered the placing of them within one phase from the mean as a perfect check in determining the months to be averaged or repeated. Should there be a greater error than this in determining the position of a maximum or a minimum it would mean that there was a slight error in the curve and that it was necessary to apply a slight adjustment factor to the values of the period taken from it. In no case have I had a large factor to apply, thereby showing that the curve as constructed was approximately correct. Indications from the work explained above are that the period taken from it can be relied upon to within three or four months and that such errors as do occur are canceled in most cases by ones of opposite sign before adjustment has become serious. Table 3 shows which months I have averaged and repeated in the analysis of the rainfall data of each State of the United States. It is probably useless to emphasize that there was no change in this table for any of the 42 sections into which the Weather Bureau has divided the United States. At first thought the results of Table 3 and of figure 8 are startling. However, an inspection of the much greater changes in the period which have persisted through entire cycles during the last 115 years, namely, from 88 to 205 months, show that these variations through short periods of time are to be expected. Moreover, there is no way to draw a curve satisfying the necessary conditions and having smaller variations, unless possibly by introducing more points of maxima and minima upon it. Such a complication would be much less probable than the variations shown by the present one, all of which variations are less than the variations from the mean value of complete cycles of approximately 11 years have been in the rather recent past, as shown by Table 1.

THE RAINFALL DATA EXAMINED.

I have examined all the rainfall averages given for each of the 42 sections of the United States in the Monthly Climatological Data published by the Weather Bureau. The dates of beginning of all section records are shown in

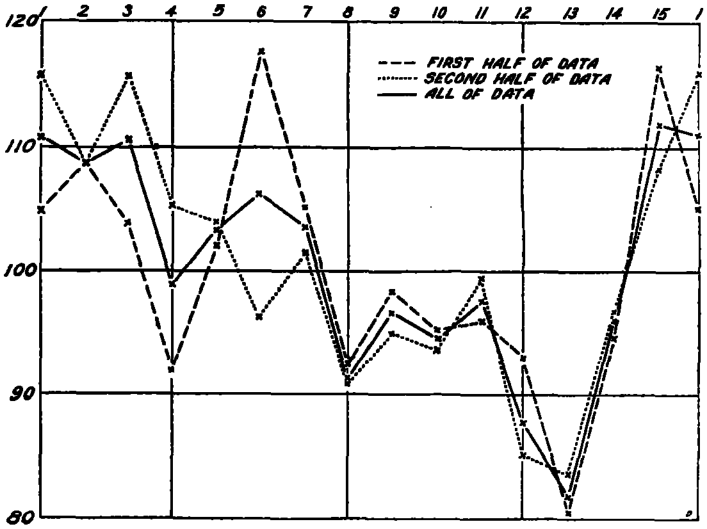


FIG. 1.—Unsmoothed rainfall variation for 26 Eastern States. Fundamental period equals one-ninth sun-spot period. Halves of data shown separately.

the tables accompanying this article. Of course, since the epoch of sun-spot minimum is always in the same numbered phase, i. e., phase 4, one section will begin its record on, say, a phase 1, another on a phase 7, etc. Care must be taken to work back, using Table 3, to get the initial data properly placed. All these data have been gathered to one sheet for each section and then tabulated as outlined above, repeating or averaging the months as named in Table 3. Of course it has been impossible to use the recent data for this work since it is necessary to have both a sun-spot maximum and minimum after the last date for which figure 8 is accurate.

The last minimum occurred in 1913, and all data since then are thus unavailable for use in examining the existence of the period. This would not be a handicap for predicting, if the period should be proved to exist, since the course of the maxima and minima could be followed from cycle to cycle by using means from a large number of sections and an extrapolation made for a cycle in advance without serious error. Indeed, in such a case it might be possible to predict the time of the next sun-spot maximum or minimum from the rainfall data.

Effect of annual cycle.—In many cases the residual left from the seasonal variation is large enough to distort the curves materially. I have therefore, except in the cases of New England and New York, where it is smallest, carefully eliminated it, no matter how large or how small. To do this I have prepared two tables for each section according to the plan previously outlined, repeating and averaging in each one the months determined by Table 3. In the first of these tables I have used the actual values of the rainfall as given in Climatalogical Data. In the second I have used instead of each January the mean of all the Januaries and so on for each month of the year. In this second table the mean monthly values were repeated or averaged exactly as in the first one to give a table entirely similar to the first table. The variation from phase to phase in this second table is, therefore, entirely the seasonal residual and contains all of it. For the average State in the country it is approximately 4 per cent each side of the normal, the rest of the seasonal variation having been damped out by the process of tabulating the incommensurable period which is being investigated. The quotients of each phase in the first table divided by the corresponding phase of the second give us the percentage of normal rainfall of that phase for the section concerned throughout all the years of the data. These percentages have been obtained and tabulated for each of the 42 sections of the United States. In no case has there been any smoothing of results other than that marked in the tables where the mean has been smoothed for purposes of diagrams in addition to those representing the unsmoothed values. All examinations of the data by the method of least squares have been based only on the unsmoothed values.

When the data of the whole country had been examined it was found that, ignoring minor variations, quite possibly due to accidental variations, there were three distinct groups in the country, which I have denominated Eastern Group, Pacific Group, and Central Group.

The Eastern Group contains 20 sections of 26 States, embracing all east of the Mississippi River and Louisiana have been included with the Central Group. The boundaries of such groups can not be sharp, and border in addition. The resemblances of Arkansas and Missouri to this group are strong enough as quite possibly to warrant their inclusion; or Illinois, which like them differs somewhat from the mean, might quite possibly States might be differently classified by different investigators.

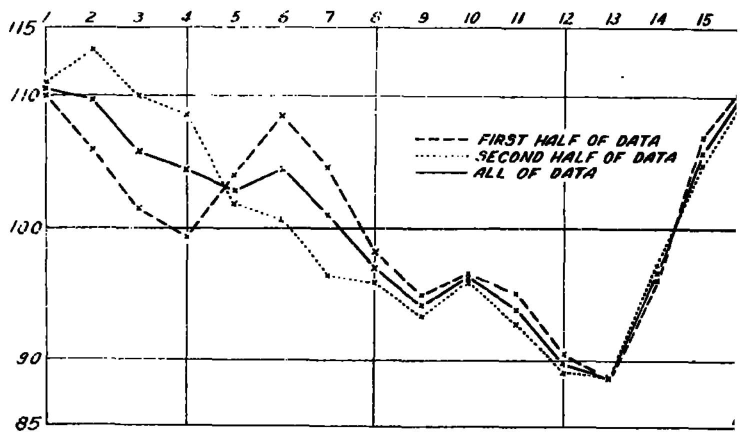


FIG. 2.—Curves of figure 1 smoothed by averaging each phase with the two immediately adjacent.

Let us examine the data of the Eastern Group as given in Table 4. Phase 13 is seen to have a rainfall less than normal for each of the 20 sections, although they extend geographically from New England to Louisiana and from Michigan to Florida. Each of the 20 has a percentage above normal for phases 15 and 1. Phases 2 and 3 are almost as unanimous. Such a distribution certainly does not in any way follow the law of probabilities of distribution of accidental errors. In Tables 5 and 6 the data from each section have been divided chronologically into halves with the exception of the two sections Maryland-Delaware and Alabama, where the whole interval of the data is not much more than the half of most of the other States. These two sections have been included in the second half of the data. Figure 1 shows the curves of each of these halves and of the whole interval as well. Figure 2 shows the same curves as figure 1, except that each has been smoothed by averaging each phase value with the two immediately adjacent. The halves are entirely independent. *The only question to settle in proving the existence of the period is whether the resemblances of the two halves can be accidental.* In phase 13 we find for the

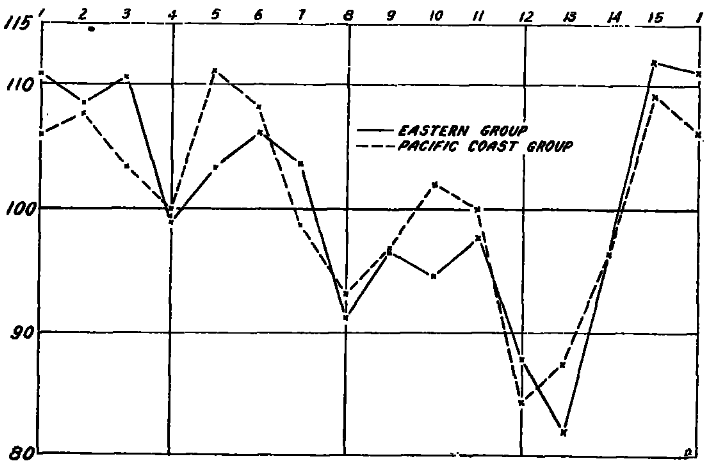


FIG. 3.—Pacific Coast Groups of nine States, both smoothed and unsmoothed. Chronologically, phase 1=phase 7 of Eastern Group.

first half of the data 1 above normal and 17 below normal; for the second half of the data, 1 above normal and 19 below normal. For the two maximum phases considered above we find 31 cases above normal and 5 below in the first half of the data and 34 above against 6 below for the second half of the data. Such resemblances seem much too striking to be accidental.

Since the differences of the halves are presumably purely accidental, we can take one-half of each of these 15 differences as a residual from the mean to determine the probable error of any point on the curve. There is no need of reproducing such routine calculation here. The result is a probable error of 2.94 per cent from the normal, despite the one large difference without which it would be less than 1 per cent. Having 15 residuals from which to determine this probable error it seems to give a good measure of the reliability of the variation. The total difference between maximum and minimum of this variation is 30 per cent of normal, or more than 10 times the probable error. The probability of an accidental error half this size is about 1 in 1,700. *Upon the acceptance of the above rests almost the whole proof of the verity of this period.* The remaining part is given merely as confirmatory and to round out a preliminary investigation of the whole of the United States.

The Pacific Group contains nine large States—Arizona, California, Oregon, Washington, Idaho, Montana, Utah, Nevada, and Wyoming. New Mexico resembles these States enough that possibly it should have been included, and Wyoming departs enough on the eastern boundary that another investigator might have included it in the Central Group. The area of the whole group is in the neighborhood of 1,000,000 square miles, about the same as that of the Eastern Group, although containing many fewer States. Each of the States was examined as in the case of the Eastern Group. At first there seemed to be a disagreement between the two sections. However, when the two curves were superposed before a window it was found that this was merely a change in phase and that there was a remarkable similarity in their shapes. I have therefore numbered as one the phase which occurs simultaneously with phase 7 of the Eastern Group.

The *seasonal* rainfall is due to solar causes modified by local conditions. In some places rain is produced during the cold months; in others during the hot months. In still others there is almost no variation from summer to winter. An analagous effect in any other period of solar origin would, therefore, not be improbable.

Table 7 shows the results for each of these States. Phase 13 has values below normal for each of the nine States. Phases 15 and 1 have 13 values above normal to 5 below. In phase 5 all values are above normal. In the Eastern Group there were 14 above and 6 below in this phase. Such a distribution of values over 1,300 miles of territory is not according to the law of probabilities of accidental errors or values.

The data of four of these States are too short to divide into halves. I have, therefore, not made a complete regional division into halves but have divided into halves only the five States with long enough records. Table 8 shows these to resemble each other quite remarkably. Once more each value of phase 13 is below normal for each half of the data. Nine of the ten values of phase 15 are above normal and only three below in each of 1 and 2. Determining the probable error, as in the Eastern Group from the 15 half differences between the unsmoothed means of the halves, we find it to be 5.74 per cent. Since in this case there are one-fourth as many sections as in the two halves of the Eastern Group, the probable error should be twice the eastern value according to the theory of probabilities, assuming an actual period such that dividing more and more sections into halves would show the halves approaching each other. It is interesting to note that half the western value is 2.87, almost identical with the 2.94 of the Eastern Group. All these results are shown graphically in figures 3, 6, and 7.

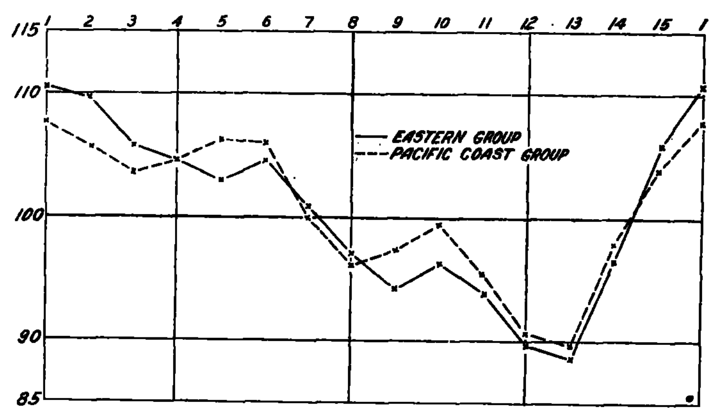


FIG. 4.—Superposition of unsmoothed curves of Eastern and Pacific Coast Groups. Chronologically, phase 1 of Pacific=phase 7 of Eastern.

Figures 5 and 4 show the superposition of the smoothed and the unsmoothed curves of the Pacific Group on the Eastern Group. The resemblance extends even to the principal minor irregularities of the two curves, indicating the same set of harmonics for each. These two curves are entirely independent of each other, since storms cross the country in a few days, while they differ in phase by approximately six months.

The Central Group contains all States which do not fall very clearly into the two groups examined above. As explained before, several of these States might easily have been included in one of the other groups to which they are contiguous. The remaining States show some common indication of a group periodicity differing by about two phases from the Eastern Group. In Table 9 the phase values are given for these States with the phases numbered as for the Eastern Group. In each of phases 14 and 1 there are 10 States below normal and only 3 above normal. Phases 2 and 3 together have 16

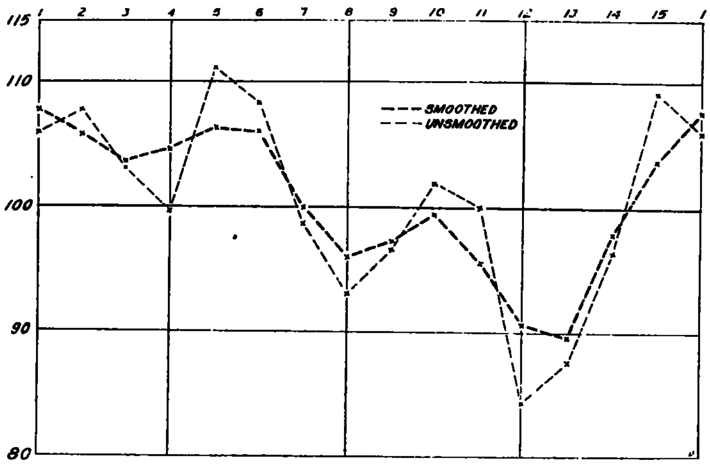


FIG. 5.—Superposition of smoothed curves of Eastern and Pacific Coast Groups. Chronologically, phase 1 of Pacific=phase 7 of Eastern.

values above normal to 10 below. Through this area there seems to be a rapid change from fair agreement with the Eastern Group as with Arkansas and Missouri to similar agreement with the Western as in New Mexico.

Prof. Dayton C. Miller, of Case School of Applied Science, has very kindly analyzed the curves of figures 1 to 5 on his harmonic analyzer. The results are shown as

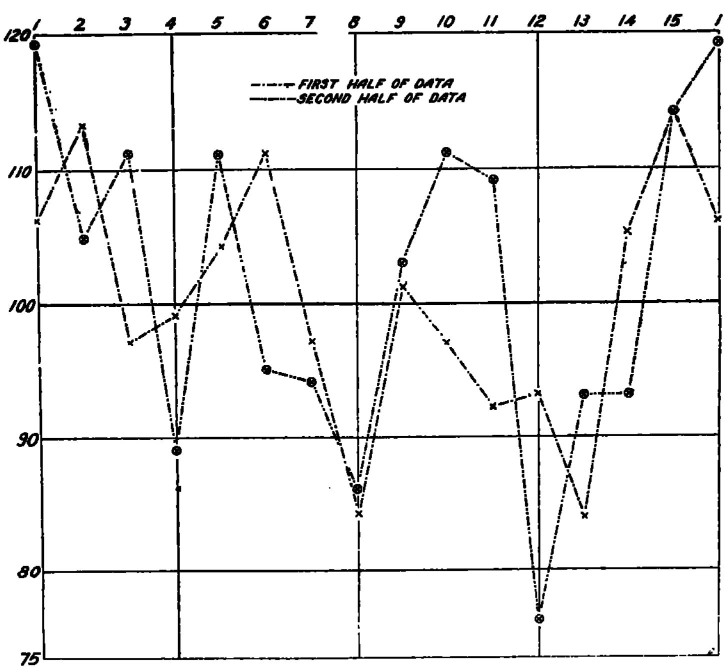


FIG. 6.—Unsmoothed halves of the data of five Western States.

Table 10. All harmonics above the fifth showed, in each case, amplitudes which were much less than the probable error of the curves as determined in an earlier part of this discussion. This indicated them as spurious as also would be shown, for harmonics above the fifth or sixth by reason of their short period with reference to the month, the unit of time for which our data are secured. The phase of each is given for the epoch of phase 1 of the various tables. The fourth harmonic has a period of approximately four months. Obviously, amplitudes and phases of this harmonic, derived from data which have the month as unit of time, must have large percentages of error. All that we can possibly secure from our analysis regarding this harmonic, or the fifth one, is a hint as to the reality of its presence. We may conclude with fair definiteness that the second and third harmonics do exist, and that their amplitudes and phases are of the order given in the tables. It is probable that the fourth harmonic exists, since its amplitudes and phases agree fairly well in the various curves. The fifth harmonic is very doubtful, so far as our evidence is concerned, its amplitudes for each of the Eastern and Pacific Groups being but little larger than the probable error. The reason for the disappearance of the fourth and fifth harmonics in the process of smoothing is, of course, self-evident.

It may occur to some that possibly there is a residual of the seasonal effect left in this period in some manner, despite the elimination explained above. There are three answers that may be given to this objection, all of which are merely the same one in different form.

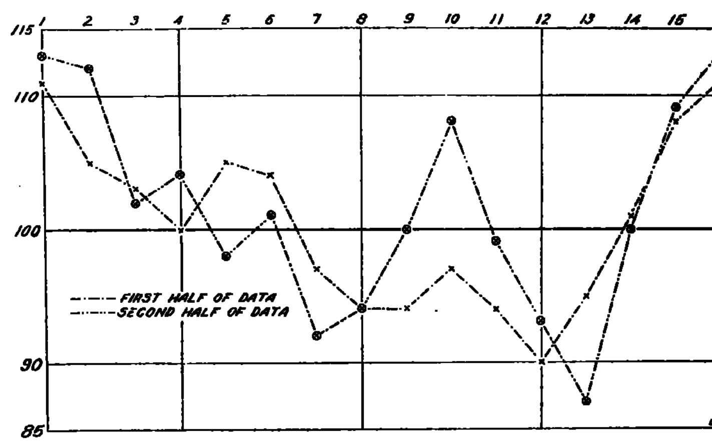


FIG. 7.—Smoothed halves of data of five Western States.

(*a*) In Prof. Schuster's discussion of the periodogram (**7**) method of searching for periods we find the following: "There is a limit beyond which it is useless to go. This limit is reached when the values of A and B for two closely adjoining values n_1 and n_2 are no longer independent of each other. The theory of vibration shows that independence begins when there is an ultimate disagreement of phase amounting to about one-quarter of a period."

(*b*) Prof. Turner has worked out the effects of any period on adjoining periods (**8**). He divides the data into integral parts and calls any one of these submultiples q; p is a period near q, such that $q+x=p. x<1$. From the Fourier sequence the periods q and $q+1$ are independent. Let us consider the seasonal period as q and the ninth harmonic of the sun-spot period as p. In order that x may be as small as 1, we must have $q=3$. That x be less, requires $q=2$. But, quoting Prof. Turner, "q is a fairly large integer for any periodicity worth serious consideration."

(*c*) The work involved in computing the periods near 12 months for each State is much greater than the value of the results. I have, however, taken Pennsylvania as typical of either the Eastern or Pacific Group and computed periods of 12, 13, 14, 15, and 16 months.

For 12 months, which is the seasonal period, the amplitude of the variation is 34 per cent; for 13 months it is 11 per cent; for 14 months it is 12 per cent; for 15 months it is 10 per cent; and for 16 months 17 per cent; the amplitude of the ninth harmonic of the sun-spot period is 26 per cent. The mean value of the ninth harmonic during this interval of years was 15.8 months, showing the increase in amplitude at the nearest of the other periods as demanded by the theory of the periodogram (**7**) or by the Fourier sequence (**10**).

If the conclusions of this paper are accepted (**11**), either as proved or as probable, it is but the beginning of a very long search. In this search it will be necessary first to examine anew all the sun-spot data by the method of figure 8. Having formed this period curve, each month's sun-spot count as gathered by Wolf and Wolfer must be adjusted to throw it in its proper phase in the cycle. This done the mean of each phase must be plotted and submitted to harmonic analysis, as has been done without such adjustment (**9**). The harmonics found to stand out most strongly must then be used as a basis for a search for other rainfall periods. Furthermore, all this work must be done for temperature variations as well, unless the first periods tried yield negative results.

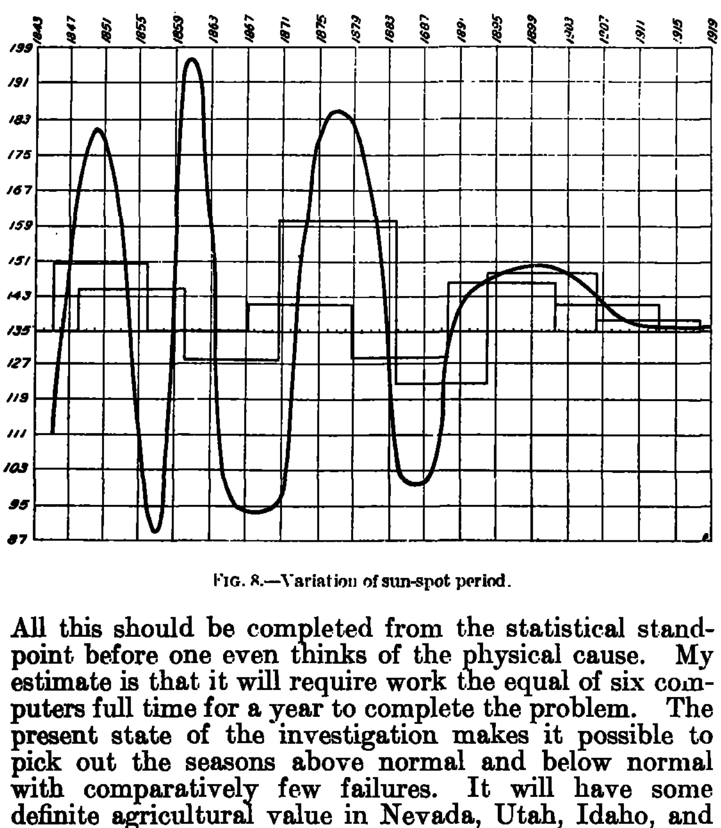


FIG. 8.—Variation of sun-spot period.

All this should be completed from the statistical standpoint before one even thinks of the physical cause. My estimate is that it will require work the equal of six computers full time for a year to complete the problem. The present state of the investigation makes it possible to pick out the seasons above normal and below normal with comparatively few failures. It will have some definite agricultural value in Nevada, Utah, Idaho, and Montana in determining what years will produce crops on land which can not be irrigated. Should one or more similar periods be found, the value of the combined result for such places would be very great.

In conclusion, I wish to thank several persons for assistance. The Research Committee of the Graduate School of the University of Kansas engaged Mr. Anthony Oates as a computer for the last part of the work. Mrs. William Thaw made it possible for me to devote my summer vacation to research. The Chief of the U. S. Weather Bureau and Mr. S. D. Flora, State Meteorologist for Kansas, made a very large amount of data available to me. Prof. Marvin has suggested many changes in wording which have been incorporated to make the meaning clearer to the readers. Prof. Dayton C. Miller made an harmonic analysis of eight of the variation curves that would have been impossible mathematically because of the tremendous amount of work involved. Prof. F. E. Kester gave a very large amount of time to discuss each phase of the problem with me. Any success that the work may have will be due in large measure to his suggestions and criticisms. Several astronomers and meteorologists to whom the manuscript was submitted for examination made suggestions which have been incorporated. To each of these I owe my sincerest thanks for without their aid the work would probably have failed.

BIBLIOGRAPHY.

(**1**) **Douglass, A. E.:** The correlation of sun spots, weather, and tree growth. *Pub. American Astronomical Soc.*, 1918, Vol. III, p. 121.

(**2**) **Hann, J.:** Sun spots and rainfall. *Handbook of Climatology* (trans. by R. DeC. Ward), Vol. I, p. 406. Brückner: *Klimaschwankungen seit 1700* (Vienna, 1890), has shown a varying period of about thirty-five years in rainfall and temperature. Lockyer: The solar activity, 1833–1900. *Proc. Roy. Soc.*, vol. 68, p. 285, showed a possible correlation of this period with sun spots. Clough, H. W.: Synchronous variations in solar and terrestrial phenomena. *Astrophys. Jour.*, vol. 22, p. 42, has greatly strengthened the evidence of such a relationship. They have also considered the variation in length of the sun-spot period. Clough says: "The solar-spot activity is periodically accelerated and retarded, and this action is primarily manifest in the varying length of the 11-year spot cycle, since it operates continuously throughout the entire interval to accelerate or retard the occurrence of the two phases" (*loc. cit.* p. 59).

(**3**) **Turner, H. H.:** A fifteen-month period in earthquakes. *Monthly Notices, Roy. Astronomical Soc.*, April, 1919, p. 461.

(**4**) **Alter, Dinsmore:** Possible connection between sun spots and earthquakes. *Science*, May 14, 1920, p. 486.

(**5**) **Wolfer, A.:** Revision of Wolf's sun-spot relative numbers. Mo. WEATHER REV., April, 1902 pp. 171–176; Tables of sun-spot frequencies, *ibid.*, July, 1915, p. 314; Tables of sun-spot frequency for the years 1902–1919, *ibid.*, Aug., 1920, p. 459.

(**6**) **Turner, H. H.:** On a simple method of detecting discontinuities in a series of recorded observations, with an application to sun spots, suggesting that they are caused by a meteor swarm due to successive encounters of the Leonids with Saturn, which has been more than once pertubed by the Leonid swarm. *Monthly Notices, Roy Astronomical Soc.*, Dec., 1913, pp. 82–109.

(**7**) **Schuster:** On the periodicites of sun spots. *Phil. Trans.* 206A (1906), p. 71.

(**8**) **Turner, H. H.:** Sun-spot periodicity as Fourier series. *Monthly Notices, Roy. Astronomical Soc.* 1913, pp. 714–732, especially p. 717.

(**9**) **Dogobert, Dr. Elsa (Frenkel):** Untersuchungen über Kurzperiodische Schwankungen der Häufigkeit der Sonnenflecken. *Publikationen der Sternwarte des eidgenossischen Polytechnikums*, Band V., 1913. Thesis in which the author investigated periods from 150 days to 24 months by Schuster's periodogram method. She secures a hint of a 14-month period, but does not carry it through as probable.

(10) **Turner, H. H.**: Further remarks on the expression of sun-spot periodicity as a Fourier sequence. *Monthly Notices, Roy. Astronomical Soc.* 1913, pp. 16–26. By means given by Turner it is sometimes, though not always, possible to determine masked periodicities.

(11) **Douglass, A. E.**: Climatic cycles and tree-growth. *Carnegie Institution of Wash.*, 1919. The author seems to have proved definitely a correlation between climate and sun spots. Nothing on this subject that has yet come to my attention begins to compare in importance with this work.

TABLE 1.—*Wolf's & Wolfer's table of sunspot maxima and minima.*

[Copied from MONTHLY WEATHER REVIEW, August, 1920.]

Epochs.	Minima weights.	Periods.	Epochs.	Maxima weights.	Periods.
1610.8	5	....	1615.5	2	....
1619.0	1	8.2	1626.0	5	10.5
1634.0	2	15.0	1639.5	2	13.5
1645.0	5	11.0	1649.0	1	9.5
1655.0	1	10.0	1660.0	1	11.0
1666.0	2	11.0	1675.0	2	15.0
1679.5	2	13.5	1685.0	2	10.0
1689.5	2	10.0	1693.0	1	8.0
1698.0	1	8.5	1705.5	4	12.5
1712.0	3	14.0	1718.2	6	12.7
1723.5	2	11.5	1727.5	4	9.3
1734.0	2	10.5	1738.7	2	11.2
1745.0	2	11.0	1750.3	7	11.6
1755.2	9	10.2	1761.5	7	11.2
1766.5	5	11.3	1769.7	8	8.2
1775.5	7	9.0	1778.4	5	8.7
1784.7	4	9.2	1788.1	4	9.7
1798.3	9	13.6	1805.2	5	17.1
1810.6	8	12.3	1816.4	8	11.2
1823.3	10	12.7	1829.9	10	13.5
1833.9	10	10.6	1837.2	10	7.3
1843.5	10	9.6	1848.1	10	10.9
1856.0	10	12.5	1860.1	10	12.0
1867.2	10	11.2	1870.6	10	10.5
1878.9	10	11.7	1883.9	10	13.3
1889.6	10	10.7	1894.1	10	10.2
1901.7	10	12.1	1906.4	10	12.3
1913.6 [1]	10	11.7	1917.6	10	11.2

[1] Table 3 has used 1913.4, the date published in the MONTHLY WEATHER REVIEW, July, 1915, 43:314. A change to the new date would slightly better the results of Tables 5 and 6 and 8. Since, however, the disagreement is a minor one, the tables and figures will not be changed on account of this later value.

TABLE 2.

Year.	Period.	Departure.	Year.	Period.	Departure.	Year.	Period.	Departure.
	Months.			*Months.*			*Months.*	
1850	180	+45	1871	106	−29	1892	144	+ 9
51	176	+41	72	135	0	93	145	+10
52	165	+30	73	156	+21	94	146	+11
53	146	+11	74	170	+35	95	147	+12
54	125	−10	75	180	+45	96	148	+13
55	100	−35	76	184	+49	97	149	+14
56	90	−45	77	184	+49	98	149	+14
57	93	−42	78	184	+49	99	149	+14
58	125	−10	79	181	+46	1900	149	+14
59	174	+39	80	173	+38	01	149	+14
60	196	+61	81	161	+26	02	148	+13
61	196	+61	82	144	+ 9	03	147	+12
62	173	+38	83	113	−22	04	146	+11
63	143	+ 8	84	102	−33	05	144	+ 9
64	104	−31	85	100	−35	06	142	+ 7
65	97	−38	86	100	−35	07	140	+ 5
66	94	−41	87	101	−34	08	138	+ 3
67	93	−42	88	108	−27	09	137	+ 2
68	93	−42	89	128	− 7	10	136	+ 1
69	94	−41	90	138	+ 3	11	136	+ 1
70	96	−39	91	142	+ 7	12	135	0

TABLE 3.—*Data repeated or averaged in keeping rainfall periodicity in-step with sunspots.*

Repeated.	Skipped or averaged.	Repeated.	Skipped or averaged.
1871, April.	1872, April. 1873, Sept. 1874, April, Oct. 1875, March, June, Nov. 1876, Feb., May, Aug., Nov. 1877, Jan., April, July, Sept., Dec. 1878, March, June, Aug., Nov. 1879, March, July, Nov. 1880, April, Oct. 1881, July.	1884, Sept. 1885, April, Oct. 1886, April, Sept. 1887, Jan., May, Sept. 1888, Jan., May, Sept. 1889, Feb., Aug.	1893, April. 1894, May. 1895, May. 1896, April. 1897, March. 1898, Jan., Dec. 1899, Dec. 1901, Jan. 1902, April. 1903, Sept. 1909, July. 1913, Jan.

TABLE 4.—*Rainfall data of Eastern Group of States, by phases.*

Section.	Phase numbers.															Total number of cycles in data.	First month and year of data.[1]
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
New England	109	106	112	103	101	95	108	80	108	98	86	96	78	104	114	20	January, 1888.
New York	106	114	115	95	88	102	101	82	106	86	87	105	86	104	118	18	January, 1890.
Pennsylvania	111	110	103	101	96	107	105	84	91	90	98	99	83	103	114	20	January, 1888.
New Jersey	110	105	118	101	93	109	102	81	103	99	96	98	80	105	103	23	January, 1885.
Maryland-Delaware	127	88	119	98	108	90	103	93	92	102	100	82	85	100	111	11	January, 1899.
Virginia	114	109	121	92	105	106	105	92	100	89	104	79	78	95	102	17	July, 1891.
West Virginia	112	105	106	93	101	115	119	96	94	89	104	82	76	97	109	17	January, 1891.
North Carolina	108	106	119	93	108	110	103	84	103	95	96	86	74	99	115	21	January, 1887.
South Carolina	108	114	115	93	109	109	101	87	97	91	100	90	78	101	114	21	January, 1887.
Georgia	110	115	113	92	109	98	97	89	99	89	100	87	86	95	116	17	October, 1891.
Florida	102	120	117	99	111	100	83	82	99	104	93	82	96	93	119	17	September, 1891.
Alabama	112	107	130	104	112	95	103	94	96	90	95	67	81	102	108	13	January, 1896.
Mississippi	111	108	113	112	112	104	89	100	92	102	100	81	78	88	112	20	January, 1888.
Louisiana	102	108	104	112	110	104	103	92	95	112	106	75	80	89	113	17	January, 1891.
Tennessee	110	102	102	106	108	110	98	102	90	97	107	80	79	99	108	24	March, 1883.
Kentucky	122	110	103	98	111	118	102	96	88	92	101	81	77	86	112	19	January, 1889.
Ohio	110	108	95	107	98	114	121	91	89	94	96	93	76	92	116	24	January, 1883
Michigan	101	113	106	89	85	107	105	94	109	90	84	101	99	98	120	20	January, 1888
Indiana	123	108	101	95	103	120	116	99	87	84	99	91	82	83	111	21	January, 1887
Illinois	110	111	96	92	99	110	110	102	96	97	101	93	90	89	105	28	January, 1878
Above	20	19	18	8	14	15	16	1	6	4	6	2	0	7	20	....	
Below	0	1	2	12	6	4	4	17	14	16	10	18	20	12	0	....	
Mean	111	108	111	99	103	106	104	91	97	94	98	88	82	96	112	....	
Smoothed	110	110	106	104	103	104	100	97	194	96	93	89	89	97	106	....	

[1] Last year for all data, 1913.

TABLE 5.—*First half of data of Eastern Group, by phases.*

Sections.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
New England	115	117	99	107	107	108	100	82	109	96	88	102	71	92	109
New York	109	125	110	88	85	105	110	86	105	93	83	106	82	94	121
Pennsylvania	106	116	101	97	99	125	104	77	103	98	84	109	72	102	110
New Jersey	110	118	110	105	96	129	94	86	97	91	94	101	77	96	95
Maryland-Delaware	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Virginia	99	120	115	88	102	125	113	83	105	83	98	85	67	95	107
West Virginia	101	98	106	87	95	114	128	94	94	98	97	96	72	102	116
North Carolina	98	108	108	85	114	131	103	81	100	87	95	95	70	105	120
South Carolina	98	115	110	89	119	120	97	89	86	85	99	101	71	105	117
Georgia	101	115	118	87	118	101	91	84	100	82	104	93	84	92	131
Florida	101	138	117	99	109	104	89	72	98	96	95	72	87	89	128
Alabama	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Mississippi	114	96	117	90	115	121	70	109	94	100	99	88	78	86	120
Louisiana[1]	90	111	111	99	113	111	94	97	97	100	112	79	96	88	110
Tennessee	107	86	93	98	104	126	102	93	92	105	115	89	82	91	118
Kentucky	113	82	93	85	107	123	111	98	102	110	94	80	84	88	124
Ohio	103	104	92	103	90	114	141	87	98	103	93	90	75	95	117
Michigan	103	104	91	78	73	118	112	105	107	87	81	103	107	98	124
Indiana	118	93	86	78	90	127	130	107	93	96	97	90	90	91	120
Illinois	102	112	88	91	103	118	108	107	93	105	101	93	83	90	107
Above normal	14	13	11	3	11	18	11	4	6	4	4	6	1	4	17
Below normal	4	5	7	15	7	0	6	14	10	12	14	12	17	14	1
Mean	105	109	104	92	102	118	105	91	98	95	96	93	80	94	116
Smoothed	110	106	102	102	104	108	105	98	95	96	95	90	89	97	105

TABLE 6.—*Second Half of rainfall data of Eastern Group by phases.*

Sections.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
New England	103	95	127	99	95	81	117	79	108	99	84	89	85	118	121
New York	103	102	121	104	92	100	92	79	108	79	92	105	91	115	116
Pennsylvania	115	105	106	107	95	89	106	89	80	83	114	92	94	104	119
New Jersey	110	90	126	96	90	88	109	77	110	107	98	95	83	114	112
Maryland-Del.	127	88	119	98	108	90	103	93	92	102	100	82	85	100	111
Virginia	131	98	127	100	108	83	97	101	94	96	111	72	88	96	96
West Virginia	122	110	105	99	110	112	105	97	92	78	113	87	80	90	99
N. Carolina	122	103	132	102	102	88	103	86	106	104	95	77	77	91	112
S. Carolina	118	110	120	96	99	95	104	83	107	98	100	77	84	96	108
Georgia	120	116	108	98	98	96	104	95	99	97	97	82	88	100	99
Florida	102	100	118	97	112	95	75	91	99	112	91	92	108	95	107
Alabama	112	107	130	104	112	95	103	94	96	90	95	67	81	102	108
Mississippi	107	119	110	135	110	87	108	91	91	103	100	74	76	88	102
Louisiana	116	105	96	126	106	96	115	87	93	123	100	72	63	88	117
Tennessee	114	119	113	114	112	96	96	112	88	90	99	73	77	99	99
Kentucky	133	140	114	112	116	111	93	95	73	73	108	83	69	84	98
Ohio	120	112	100	111	108	114	97	96	79	84	100	97	75	90	114
Michigan	99	122	120	100	98	95	97	82	100	93	86	99	91	99	117
Indiana	128	125	119	115	117	113	99	94	80	71	102	94	74	73	101
Illinois	118	110	104	92	94	102	111	98	96	89	102	94	96	87	103
Above normal	19	15	18	10	12	5	12	2	6	6	6	1	1	5	15
Below normal	1	4	1	8	8	14	8	18	14	14	9	19	19	13	5
Mean	116	109	116	105	104	96	102	91	95	94	99	85	83	96	108
Smoothed	111	114	110	108	102	101	96	96	93	96	93	89	88	96	107

TABLE 7.—*Rainfall data of Pacific Group of States by phases.*

Sections.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total number of cycles in record.	First month and year of data.[1]
Arizona	112	87	102	91	112	109	111	129	99	88	96	72	89	114	88	13	1897 January.
California	84	137	122	118	115	127	93	100	111	101	116	78	73	69	101	13	1897 January.
Nevada	140	141	105	91	109	93	80	87	98	98	87	73	87	92	130	19	1889 March.
Oregon	92	105	106	102	108	109	99	84	92	118	109	82	90	92	114	18	1890 January.
Washington	92	96	102	92	104	118	106	84	78	106	112	82	88	106	125	18	1890 January.
Idaho	107	109	109	82	116	110	106	95	81	96	109	89	84	98	118	16	1893 May.
Montana	131	92	87	83	119	105	115	98	87	92	111	98	98	88	95	14	1895 January.
Utah	123	99	101	94	115	99	103	87	95	96	95	81	86	111	112	17	1892 January.
Wyoming	120	100	104	93	107	109	103	83	99	98	98	104	89	98	101	17	1892 January.
Above normal	6	4	8	2	9	7	6	1	1	3	5	1	0	3	7	....	
Below normal	3	4	1	7	0	2	3	7	8	6	4	8	9	6	2	....	
Mean	111	107	104	94	112	109	102	94	93	99	104	84	87	96	109	....	
Smoothed	109	107	102	103	105	108	102	96	95	99	96	92	89	97	105	....	

NOTE.—Phase 1 of the Pacific Group occurs simultaneously with phase 7 of the Eastern.

[1] Last year for all data 1913.

TABLE 8.—*Halves of rainfall data for Oregon, Washington, Utah, Nevada, and Wyoming, by phases.*

[Not enough data to divide from other four States of this group.]

FIRST HALF.

Section.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Oregon	95	128	99	111	120	118	93	82	96	115	94	69	85	93	105
Washington	102	118	96	101	109	109	104	84	82	112	113	74	90	94	98
Utah	100	104	94	99	100	100	115	84	100	78	91	104	79	131	120
Nevada	129	116	95	91	95	113	70	89	109	93	69	99	79	102	145
Wyoming	105	101	103	94	94	114	101	82	116	88	92	120	87	106	104
Mean	106	113	97	99	104	111	97	84	101	97	92	93	84	105	114
Smoothed	111	105	103	100	105	104	97	94	94	97	94	90	95	101	108

SECOND HALF.

Section.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Oregon	87	84	113	93	92	95	106	82	86	120	124	95	95	90	129
Washington	78	80	114	84	98	116	98	94	84	110	114	100	94	120	127
Utah	143	94	108	93	125	98	89	91	88	118	98	54	93	88	102
Nevada	148	166	114	86	122	63	72	80	95	100	104	46	92	80	111
Wyoming	137	101	106	91	119	104	105	82	82	108	106	87	90	89	100
Mean	119	105	111	89	111	95	94	86	103	111	109	76	93	93	114
Smoothed	113	112	102	104	98	101	92	94	100	108	99	93	87	100	109

TABLE 9.—*Rainfall data of Central Group, by phases, numbered the same as Eastern Group.*

Section.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total number of cycles in record.	First year and month of data.[1]
Arkansas	117	110	102	101	111	94	82	90	94	99	116	85	91	83	112	17	1891, January.
Missouri	108	105	101	101	101	113	101	108	92	94	100	99	89	92	103	20	1888, January.
Iowa	92	102	95	97	102	100	109	113	99	90	97	104	104	80	87	18	1890, January.
Wisconsin	96	117	103	100	89	112	106	97	119	88	101	98	103	96	74	17	1891, January.
Minnesota	98	114	98	97	99	114	109	115	114	84	105	91	113	91	74	17	1891, January.
North Dakota	99	106	102	104	106	90	102	117	107	88	111	104	108	94	64	17	1891, November.
South Dakota	108	102	103	102	103	116	110	109	93	104	97	99	99	84	70	18	1890, January.
Nebraska	97	88	104	110	109	108	98	109	101	101	95	109	92	85	102	29	1876, January.
Kansas	86	88	112	100	100	110	92	123	105	92	102	104	89	95	94	21	1887, January.
Oklahoma	96	89	93	89	96	101	82	106	108	102	105	113	82	103	132	16	1893, January.
Texas	111	115	87	98	90	93	86	97	125	122	92	90	97	82	112	17	1891, January.
New Mexico	94	95	90	90	107	118	80	95	116	108	95	99	94	109	103	17	1892, January.
Colorado	89	97	107	87	99	105	106	82	110	98	106	96	95	108	114	20	1888, January.

[1] Last year for all data, 1913.

TABLE 10.—*Harmonic analysis of rainfall curves; unsmoothed amplitudes.*

	Harmonics.				
	First.	Second.	Third.	Fourth.	Fifth.
PER CENT OF NORMAL FROM MINIMUM TO MAXIMUM.					
Eastern Group, whole data	17.2	8.8	10.0	5.2	4.2
First half data, Eastern Group	13.6	9.6	13.6	6.2	10.0
Second half data, Eastern Group	21.0	11.0	6.8	6.7	0.6
Pacific Group	14.0	4.7	13.0	4.3	3.2
UNSMOOTHED PHASES AT EPOCH OF PHASE 1 OF CURVES.					
Eastern Group, whole data	39°	83°	114°	251°	234°
First half data, Eastern Group	27	118	105	273	236
Second half data, Eastern Group	47	57	127	219	323
Pacific Group	27	78	138	303	303
SMOOTHED AMPLITUDES, PER CENT OF NORMAL FROM MINIMUM TO MAXIMUM.					
Eastern Group, whole data	16.1	6.8	5.6	1.6	0.2
First half data, Eastern Group	13.0	7.3	8.1	1.6	0.4
Second half data, Eastern Group	19.8	8.4	3.6	1.5	0.0
Pacific Group	13.2	4.0	7.0	1.3	0.0
SMOOTHED PHASES AT EPOCH OF PHASE 1 OF CURVES.					
Eastern Group, whole data	38°	85°	113°	238°	102°
First half data, Eastern Group	26	118	105	278	302
Second half data, Eastern Group	48	57	128	222	207
Pacific Group	30	80	138	309	313

DISCUSSION.

By C. F. MARVIN.

If we understand Mr. Alter's claims correctly, he embraces the idea advocated by Mr. Clough, namely, that the duration of the sun-spot period is variable; that is, it is systematically lengthened and shortened. With this principle as a basis in conjunction with Wolfer's values of the epochs of sun-spot maxima and minima and by means of a graphic integration represented by his diagram, figure 8, Mr. Alter arrives at the highly variable values of the length from year to year of the sun-spot period beginning about 1847. One-ninth of this period, stated in months, then, becomes the variable length of the alleged cycle in rainfall. By methods, details of which are made clear, the rainfall data of the Weather Bureau for practically the entire United States are analyzed, and Mr. Alter seems convinced that he has hit upon a very important period or cycle, both in sun-spot numbers and also in rainfall sequences in the United States. Acceptance of Mr. Alter's conclusions at once commits one to his claim that he has established as more or less probable that sun spottedness or some related solar activity is at least one factor in the control of United States rainfall.

In order that the reader may be spared any uncertainty of mind, the writer may say frankly, at the outset of this discussion, that he is convinced that little if any thing at all as to a cycle in rainfall or a connection between rainfall and sun-spots is proved by the investigation.

The discussion may proceed under the following topics:

(1) The proposition is irrational.

(2) The quantitative basis of figure 8 (variable length of sun-spot period) is hypothetical and inadequate.

(3) The method of layout of data and computation of results introduces glaring sources of error and uncertainty.

(4) Least square methods, in so far as they are brought to bear on the problem, have a limited significance.

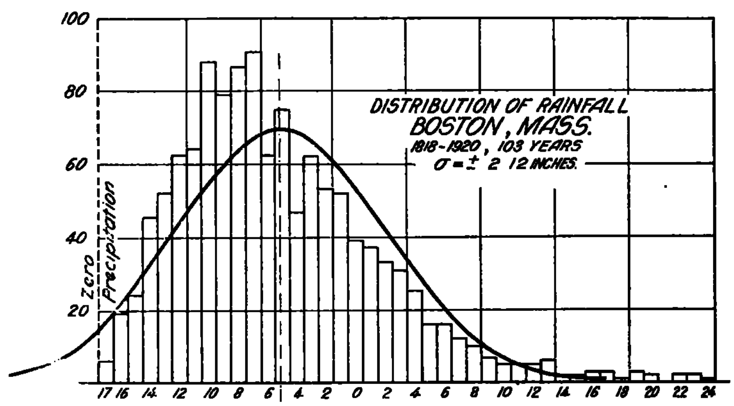
(5) The rainfall data are seemingly heterogeneous.

Only a very brief discussion of these topics is now possible.

(1) *The proposition is irrational.*—A great deal is already known with reference to the definite physical causes of rainfall, its distribution both as to continental area and as to topography, also as to time and the march of seasons. We may fairly say that practically every feature of the occurrence of precipitation, especially within the United States, is intimately associated with the general circulation of the atmosphere and the sequences of cyclones and anticyclones. How can we be convinced that the features which appear in Mr. Alter's results are not very largely or entirely caused by the uneliminated features of rainfall dependent upon the general circulation of the air?

The simple method of tabulation of highly composite data in columns employed by Mr. Alter can not be admitted to exclude and otherwise wholly eliminate extraneous influences, except, possibly, when the number of observations is very great, and even then it must be demonstrated a systematic residual from one cause or another is not included.

There can not be anything unique or magical in a changeable period of time, which shall constantly be



one-ninth of a hypothetically changing sun-spot period. Other integral fractional parts corresponding to the remaining 8 digits, as also many other multiples and submultiples, have an equal claim on our imaginations and on the probabilities and possibilities of the situation. If the reality of any one of these is admitted, on what basis can the others be rejected, and what is the consequence of the acceptance of all? This line of thought leads exactly to the same consequences as when we recognize that any succession of variable values can be *represented* more or less exactly by a Fourier series. It may be demonstrated that the original data are the summation of the several component elements into which they may be analyzed, but this is of no significance whatever as indicating the real physical existence of any or all of the components.

Of course, science is either inductive or deductive. While the absence of an entirely rational cause or explanation of certain assumed or suspected relations does not justify rejection of the hypothesis, nevertheless, on the other hand, purely inductive results, or fragments of results, without a basis of rationality, must necessarily be viewed with skepticism, or their physical reality must be demonstrated by incontrovertible proofs.

(2) *Quantitative basis of figure 8 is inadequate.*—Probably every student of sun-spot data has recognized the